FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS | **MAXIMUM MARKS: 100**

NOTE: (i) Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B. ALL** questions carry **EQUAL** marks.
(iii) Extra attempt of any question or any part of the attempted question will not be considered.
(iv) **Use of Calculator is allowed.**

SECTION-A

Q.No.1. (a) For any integer n let $a_n : Z \to Z$ by such that $a_n(m) = m+n,\ m \in Z$. Let $A=\{ a_n ; n \in Z\}$. Show that A is the group under the usual composition of mappings. **(10)**

(b) Show that the group of all inner automorphisms of a group G is isomorphic to the factor group of G by its center. **(10)**

Q.No.2. (a) Let A and B be cyclic groups of order n. Show that the set Hom(A,B) of all homomorphisms of A to B is a cyclic group. **(10)**

(b) Prove that group G is abelian iff G/Z(G) is cyclic, where Z(G) is Centre of the group. **(10)**

Q.No.3. (a) Define the dimension of a vector space V, prove that all basses of a finite dimension vector space contain same number of elements. **(10)**

(b) Show that the vectors (3, 0, -3), (-1, 1, 2), (4, 2, -2) and (2, 1, 1) are linearly dependent. **(10)**

Q.No.4. (a) The set $\{v_1, v_2, \ldots, v_n\}$ of vectors is a vector space V is linearly dependent if and only if some v_i is the linear combination of the other vectors. **(10)**

(b) Let A, B be two ideals of the ring R. Then show that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$. **(10)**

Q.No.5. (a) If A is n x n matrix then **(10)**
  (i) Determinant of $(A-\lambda I)$ where λ is a scalar in a polynomial $P(\lambda)$.
  (ii) The eigenvalues of A are the solutions of $P(\lambda) = 0$.

(b) If A is an ideal of the ring R with unity such that $1 \in A$, then $A=R$ **(10)**

SECTION-B

Q.No.6. (a) Find an equation of the straight line joining two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angles are given. Hence find equations of the tangent and normal at any point 'θ' on the ellipse. **(10)**

(b) prove that an equation of the normal to the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $x \sin t - y \cos t + a \cos 2t = 0$, t being parameter. **(10)**

Q.No.7. (a) Show that the pedal equation of the curve $x = 2a \cos\theta - a \cos 2\theta,\ y = 2a \sin\theta - a \sin 2\theta$ is $9(r^2 - a^2) = 8p^2$ **(10)**

(b) Find the length of the arc of the curve $x = e^{\theta} \sin\theta,\ y = e^{\theta} \cos\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$. **(10)**

Q.No.8. (a) Find the shortest distance between the straight line joining the points A(3, 2, -4) and B(1, 6, -6) and the straight line joining the points C(-1, 1, -2) and D(-3,1, -6). Also find equation of the line of shortest distance and coordinates of the feet of the common perpendicular. **(10)**

(b) Find an equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,\ 2x + 3y - 4z - 8 = 0$ is a great circle. **(10)**

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