RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2015

## APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED:
THREE HOURS

## MAXIMUM MARKS: 100

NOTE:(i) Attempt FIVE questions in all by selecting TWO questions from SECTION-A, ONE question from SECTION-B and TWO questions from SECTION-C. ALL questions carry EQUAL marks.
(ii) Candidate must write Q.No. in the Answer Book in accordance with Q.No. in the Q.Paper.
(iii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iv) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(v) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(vi) Extra attempt of any question or any part of the attempted question will not be considered.
(vii) Use of Calculator is allowed.

## SECTION-A

Q. No. 1. (a) Solve the initial value problem.

$$
\begin{equation*}
\frac{d y}{d x}+\frac{y}{2 x}=\frac{x}{y^{3}}, y(1)=2 \tag{10}
\end{equation*}
$$

(b) Solve $y^{\prime \prime}-4 y^{\prime}+4 y=e^{2 x}$
Q. No. 2. Solve the following equations:
(a) $\left(1-x^{2}\right) \frac{d^{2} y}{d x^{2}}-2 x \frac{d y}{d x}+2 y=0$
(b) $\frac{d^{3} y}{d x^{3}}+\frac{d y}{d x}=\operatorname{cosec} x$
Q. No. 3. (a) Classify the following:
(i) $\quad x^{2} U_{x x}+\left(a^{2}-y^{2}\right) U_{y y}=0 \quad, \quad-\infty<x<\infty,-a<y<a$
(ii) $U_{x x}-6 U_{x y}+9 U_{y y}+3 y=0$
(b) Solve

$$
\begin{align*}
& \frac{\partial^{2} u}{\partial^{2} t}=\frac{\partial^{2} u}{\partial^{2} x} \quad 0<x<5  \tag{10}\\
& u(0, t)=u(5, t)=0 \\
& u(x, 0)=x^{2}(x-5) \\
& u_{t}(x, 0)=0
\end{align*}
$$

## SECTION-B

Q. No.4. (a) Prove that if $A_{i}$ and $B_{j}$ are two first order tensors, then their product $A_{i} B_{j}(i, j=1,2,3)$ is a second order tensor.
(b) If $\phi\left(x_{1}, x_{2}, x_{3}\right)$ is a scalar point function then $\frac{\partial \phi}{\partial x_{i}}$ are the components of a first order tensor.
(c) Find the invariant of the following second order tensor

$$
\left[\begin{array}{ccc}
2 & 4 & -1  \tag{6}\\
6 & -7 & 10 \\
3 & -4 & 6
\end{array}\right]
$$

## APPLIED MATHEMATICS, PAPER-II

Q. No. 5. (a) Verify that the transformation

$$
\begin{aligned}
& x_{1}^{\prime}=\frac{1}{15}\left(5 x_{1}-14 x_{2}+2 x_{3}\right) \\
& x_{2}^{\prime}=-\frac{1}{3}\left(2 x_{1}+x_{2}+2 x_{3}\right) \\
& x_{3}^{\prime}=\frac{1}{15}\left(10 x_{1}+2 x_{2}-11 x_{3}\right)
\end{aligned}
$$

Is orthogonal and right handed. A vector field $\vec{A}$ is defined in the system

$$
O x_{1} x_{2} x_{3} \text { by } A_{1}=x_{1}^{2}, A_{2}=x_{2}^{2}, A_{3}=x_{3}^{2}
$$

Evaluate the components $A_{j}^{\prime}$ of the vector field in the new system $O x_{1}^{\prime} x_{2}^{\prime} x_{3}^{\prime}$.
(b) Prove that any second order tensor $A_{i j}$ can be written as the sum of a deviator
and an isotropic tensor.
(c) If $a_{i j}=a_{j i}$ are constants. Calculate.

$$
\frac{\partial^{2}}{\partial X_{k} \partial X_{m}}\left(a_{i j} X_{i} X_{j}\right)
$$

## SECTION-C

Q. No. 6. (a) Find the real root of the equation by using Newton - Raphson's method.

$$
\begin{equation*}
3 x-\cos x-1=0 \tag{10}
\end{equation*}
$$

(b) Solve the following system of equations by Gauss-Seidel method.

Take initial approximation as $x_{1}=0, x_{2}=0, x_{3}=0$. Perform 3 Iterations.

$$
\begin{align*}
& 20 x_{1}+x_{2}-2 x_{3}=17  \tag{10}\\
& 3 x_{1}+20 x_{2}-x_{3}=-18 \\
& 2 x_{1}-3 x_{2}+20 x_{3}=25
\end{align*}
$$

Q. No. 7. (a) Find the real root of the equation $x^{3}-4 x-9=0$ by Regular falsi method. Take the interval of the root as $(2,3)$ and perform 4 iterations. .
(b) Find a polynomial which possess through the following points:

| $x:$ | -1 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| $f(x):$ | 2 | 1 | 2 | 5 |

Q. No. 8. (a) Use the langrage's Interpolation formula to find the value $f(12)$ if the values of $x$ and $f(x)$ are given below

| $x:$ | 5 | 7 | 11 | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 150 | 392 | 1452 | 2366 |

(b) Evaluate $\int_{0}^{3} x \sqrt{1+x^{2}} d x$ using $\frac{1}{3}$ Simpson's rule and trapezoidal rule for $n=6$

